And, Or, Not

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And, Or, and Not

Let P and Q be statements.

P **and** Q is a new statement that is True if **both** P and Q are True; and false otherwise.

P **or** Q is a new statement that is True if **either** P or Q, or **both**, are True; and false otherwise.

Not P is a new statement that is True if P is False, and False if P is Q.

And

P and Q can be written $P \wedge Q$ (compare with set intersection).

$$\overset{\cap}{P \wedge Q} \qquad P \text{ and } Q$$

$$\overset{\cup}{P \vee Q} \qquad P \text{ or } Q$$

OR

P or Q can be written $P \vee Q$ (compare with set union)

Not

Not P can be written $\sim P$, or sometimes $\neg P$.

$\sim P$ $\qquad\qquad$ $\neg P$

Examples

Write the open sentences $x \neq y$ and $y \geq x$ as P and Q, P or Q, or not P.

$P(x,y) : x = y$ open sentence true if $x=y$, false if $x \neq y$

$\neg P(x,y) :=$ true if $x \neq y$, false if $x = y$

$y \geq x$ $P(x,y) : y = x$

$Q(x,y) : y > x$

$y \geq x$ " $P(x,y)$ OR $Q(x,y)$

$R(x,y)$ $y < x$

$\neg R(x,y)$ $y \geq x$

Example

Express the following in the form $P \wedge Q$, $P \vee Q$ or $\sim P$.

$\{a \mid a \neq 12\}$

R(A): $A \in \{X \in \mathcal{P}(\mathbb{N}) : |\overline{X}| < \infty\}$

$X \in \wp(\mathbb{N})$ means X is a subst of $\mathbb{N}$

like $X = \{1, 2, 3\}$ or $X = \{\text{even numbers}\}$

$X = \{a \in \mathbb{N} \mid a \neq 12\}$ $\qquad \overline{X} = \{12\}$

$|\overline{X}| = 1$

$P(A)$: $A \in \wp(\mathbb{N})$ (or $A \subseteq \mathbb{N}$)

$Q(A)$: $|\overline{A}| < \infty$

$R(A) = P(A) \wedge Q(A)$

Truth Tables

Truth tables are an effective way to keep track of combinations of statements.

P	Q	P and Q
T	T	T
T	F	F
F	T	F
F	F	F

OR

P	Q	P or Q
T	T	T
T	F	T
F	T	T
F	F	F

NOT

P	$\neg P$
T	F
F	T

$x \leq y$: $P(x,y)$ OR $Q(x,y)$

$P(x,y)$: $x < y$ $\qquad x \not\leq y$

$Q(x,y)$: $x = y$

NOT $(P(x,y)$ OR $Q(x,y))$

P	Q	P or Q	$\neg(P \text{ or } Q)$	$\neg P$	$\neg Q$	$\neg P$ and $\neg Q$
T	T	T	F	F	F	F
T	F	T	F	F	T	F
F	T	T	F	T	F	F
F	F	F	T	T	T	T

$\neg(P \text{ and } Q)$